

Artificial Intelligence

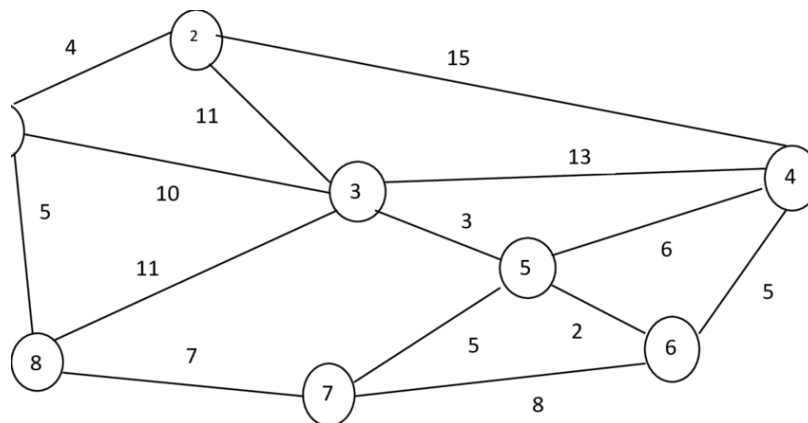
Practical Sheet 5: Genetic Algorithms

1. **Travelling Salesman Problem.** Consider an undirected graph, given by its adjacency matrix, and representing a group of cities (vertices) and the roads between them (edges). Write an evolutionary algorithm that searches for the shortest route between the N cities in the map. Use an encoding method such that the chromosomes consist of lists of integers determining the indices of the cities. Examples of five-city paths starting in city 4 are $(4,3,1,2,5)$, $(4,1,5,2,3)$ and $(4,5,1,2,3)$.

In the above example, e.g., the first chromosome thus encodes the path $4 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 5 \rightarrow 4$.

The fitness should be taken as the inverse of the route length (calculated using the ordinary cartesian distance measure, i.e. not the city-block distance measure).

The program should always generate syntactically correct routes, i.e. routes in which each city is visited once and only once until, in the final step, the tour ends by a return to the starting city. Note that specialized operators for crossover and mutation are needed in order to ensure that the paths are syntactically correct.



2. **Selection Population 1.** Consider the simple genetic algorithm applied on a population of 8 integer numbers. Suppose that at time t of the evolution the population has the following composition:

$$x=1: 2 \text{ copies}; \quad x=2: 3 \text{ copies}; \quad x=3: 3 \text{ copies}$$



Assuming that the fitness function is $f(x)=x^2$, calculate the probability of selecting the individuals $x=1$, $x=2$, and $x=3$ using roulette wheel selection.

3. **Selection Population 2.** Consider a population consisting of five individuals with the fitness values (before ranking) $f_1 = 5$, $f_2 = 7$, $f_3 = 8$, $f_4 = 10$, $f_5 = 15$. Compute the probability that individual 4 will be selected (in a single selection step) with:
 - a. Roulette wheel selection;
 - b. Roulette wheel selection, based on linearly ranked fitness values, where the lowest fitness value is set to 1 and the highest fitness value set to 10
4. **Selection Population 3.** Consider a population containing four individuals with chromosomes 101010, 000111, 010101, and 011011, and fitness values 1, 2, 3 and 4 respectively. In a given selection step, assume that individual 1 (with chromosome 101010) has been selected (using roulette-wheel selection) as the first parent. What is the probability that the schema 10xxxx will be represented in either of the two individuals resulting from the selection of a second parent, followed by crossover? (Crossover may occur, with equal probability, at any of the five available crossover points).
5. **Selection Population 4.** Consider a generational GA that has a population size of 100 individuals and uses roulette-wheel selection. Suppose that after running for t generations, the mean population fitness is 76.0 and that in the population there is only one copy of the best member, which has fitness 157.0. Also suppose that parents parent selection is performed on the population
 - a. What is the expected number of copies of the best individual in the set of selected parents?
 - b. What is the probability that there will be no copies of the best individual in the selected parents?
6. **Mutation.** Calculate the probability that a binary chromosome of length L will not be changed by applying the usual bit-flip mutation with probability $p_m = 1/L$