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Artificial Intelligence

Practical Sheet 4: Optimization

1. **Management 1.** A company manufactures 3 products "P_A", "P_B" and "P_C", seld respectively by €14, €15 and € 22 per unit. For the manufacturing of these products four types of raw materials are required ("M_A", "M_B", "M_C" and "M_D"). The prices of raw materials, the raw material units needed for each product type and the corresponding available quantities within a certain time period are included in the following table.

Material	Composition in Product	Stock
M_A	(P_A, P_B, P_C)=(0, 2, 3)	70
M_B	(P_A, P_B, P_C)=(4, 2, 1)	300
M_C	(P_A, P_B, P_C)=(3, 3, 5)	120
M_D	(P_A, P_B, P_C)=(0, 0, 1)	120

   a. The company's goal is to determine the quantities of each product that should be produced, in order to achieve the highest income. Define in detail the decision variables and form the objective function and all constraints of the problem.

2. **Management 2.** A factory makes three components, "C_A", "C_B" and "C_C" using the same production process for each. A unit of C_A takes 1 hr, a unit of C_B takes 0.75 hrs and a unit of P_C takes 0.5 hrs. In addition, C_C has to be hand finished, an activity taking 0.25 hrs per unit. Each week total production time (excluding hand finishing) must not exceed 300 hrs and hand finishing must not exceed 45 hrs.
   The components are finally assembled to make two products. One product consists of 1 unit of C_A and 1 unit of C_C, which is sold for 30€, whilst the other consists of 2 units of C_B and 1 unit of C_C and sells for 45€. At most 130 of the first product and 100 of the second product can be sold each week. Formulate the problem of planning weekly production to maximise total proceeds as a linear programming problem in 2 variables and obtain the solution graphically.

3. **Human Resources.** A company of canoes employs 120 employees, each of whom works 30 hours/week. Half of them work in the carpenter department, 20 in the plastics department, and the rest of them at the completion department. The company manufactures the simple canoes with net unit profit € 7 and the luxury canoes with corresponding profit € 10. A simple canoe requires 4.5 hours in the

carpenter department and two hours in each of the other two departments. The working hours for each luxury canoe are 5, 1 and 4 at the carpenter department, plastics department and completion department respectively. Marketing calculations have shown that not less than 1/3 and not more than 2/3 of the total number of the canoes should be luxurious.

4. Solve the following problems graphically.

   a. maximise $z = x_1 + x_2$

      subject to:

      $$x_1 + 2x_2 \leq 10$$
      $$2x_1 + x_2 \leq 16$$
      $$-x_1 + x_2 \leq 3,\ x_1 \geq 0, x_2 \geq 0.$$

   b. minimise $z = x1 + 3x2$

      subject to:

      $$x_1 + 2x_2 \geq 6$$
      $$x_1 - x_2 \leq 3$$
      $$x_1 \geq 0, x_2 \geq 0.$$