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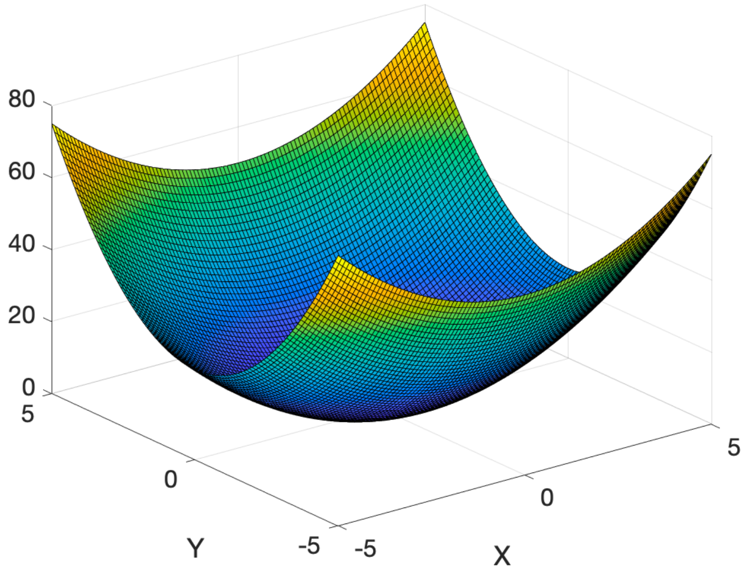
Ficha Prática 4

Gradient Descent

1. Consider the following 2D function:

$$f(x,y) = x^2 + 2y^2$$



a. Using "matplotlib" Python library, plot the function in the [-5.0, 5.0] x [-5.0, 5.0] interval.

b. Obtain the partial derivatives of *f* with respect to *x* and *y*

c. Create a Python script that, starting from a random point (x_0, y_0), uses the gradient descent algorithm to find the function minimum. This script should have two important hyperparameters:

- γ: Learning rate. (Adjusts the strength of the weights optimization step)
- δ: Patience. (Stops the learning process, when after *10* epochs, the value of *f()* does not decreased at least δ.

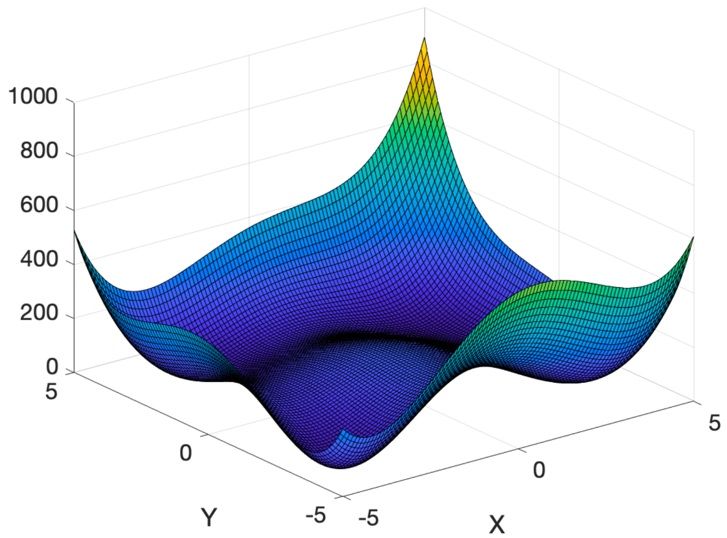
d. Using "quiver" plots, show the path from (x_0, y_0) -> (x_t, y_t) (t=number of iterations).

e. Compare the average minimum obtained (ζ) (after 100 trials), with respect to γ and δ.

f. Create a 3D plot, that relates ζ (dependent variable) to γ and δ (independent variables).

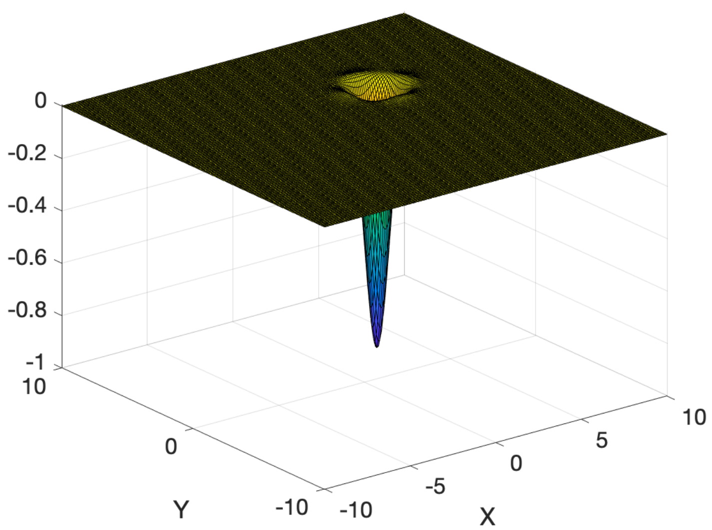
2. Repeat the 1) exercise, considering the Himmelblau's function (using [-5, 5] x [-5, 5] interval).

$$f(x, y) = (x^2 + y - 11)^2 + (y^2 + x - 7)^2$$



3. Repeat the 1) exercise, considering the Easom's function (using [-10, 10] x [-10, 10] interval).

$$f(x, y) = -\cos(x)\cos(y)\exp\left(-\left((x-\pi)^2 + (y-\pi)^2\right)\right)$$



4. Finally, repeat the 1) exercise, considering the Ackley's function (using [-5, 5] x [-5, 5] interval).

$$f(x,y) = -20\ \exp\left(-0.2\sqrt{\frac{x2+y2}{2}}\right) - \exp\left(\frac{\cos(2\pi x)+\cos(2\pi y)}{2}\right) + \exp(1) + 20$$