COMPUTER VISION
MEI/1

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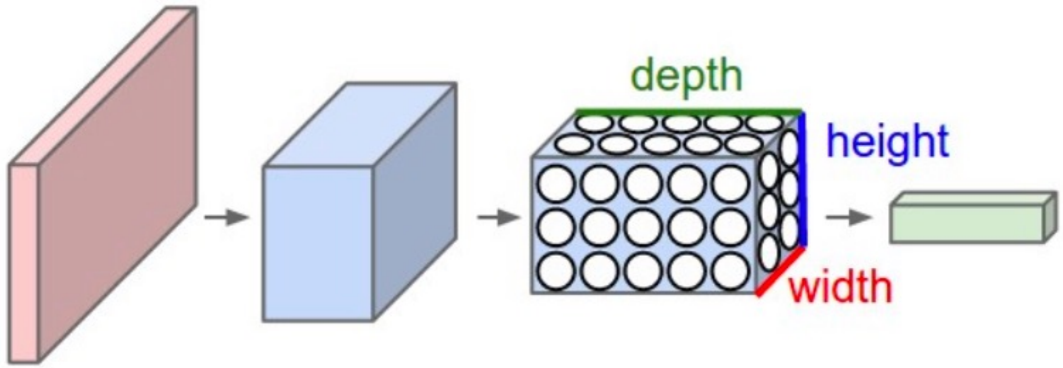
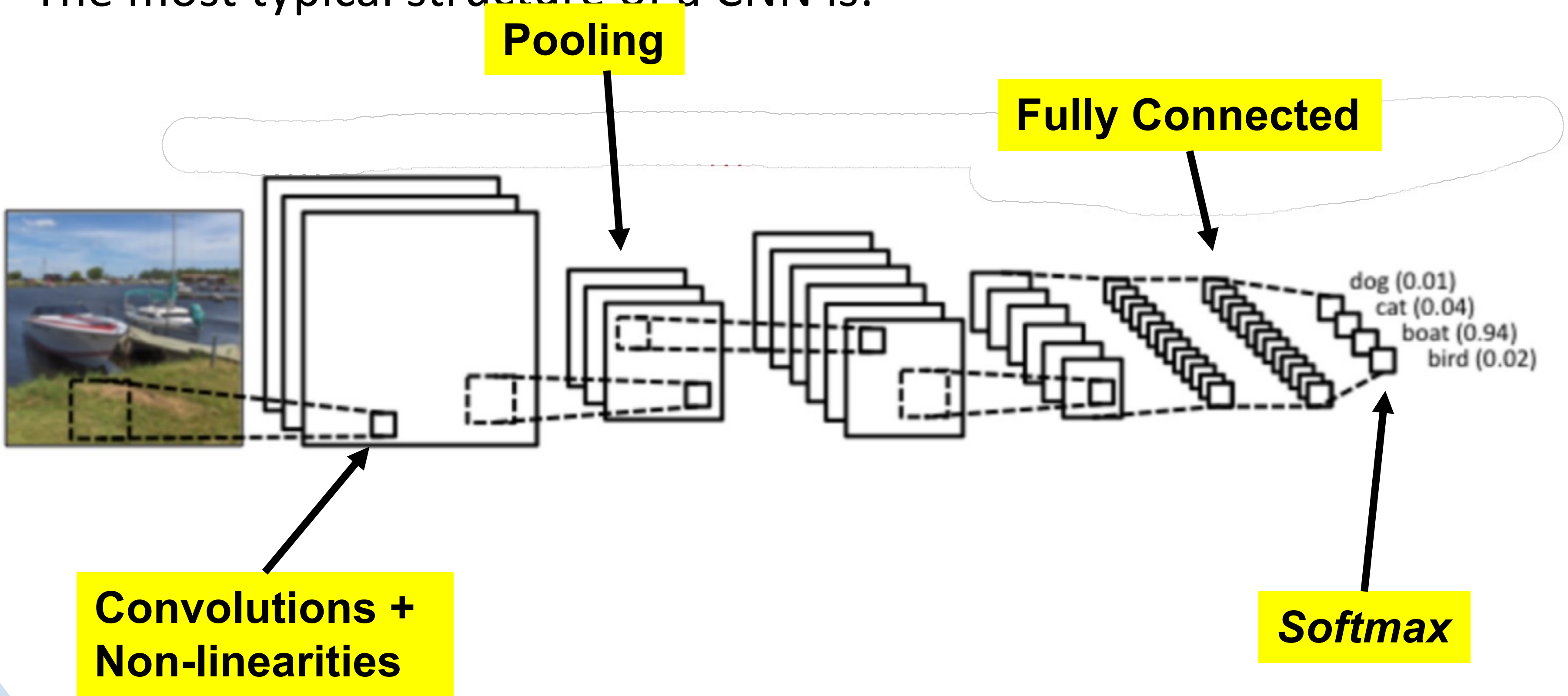
Convolutional Neural Networks (CNNs)

- CNNs are "the" type of Neural Networks that have been augmenting their popularity in most tasks related to Computer Vision
  - E.g., Object Detection, Segmentation and Classification.
- The property of shift invariance gives them the biological inspiration of the human visual system and keeps the number of weights relatively small, making learning a feasible task.
- In opposition to traditional feed-forward nets, neurons in CNNs are arranged in three dimensions.



Convolutional Neural Networks (CNNs)

- The most typical structure of a CNN is:



These operations are the basic building blocks of *most CNNs*, so understanding how these work is an important step to understand the functioning of these powerful models.

Signals and Systems

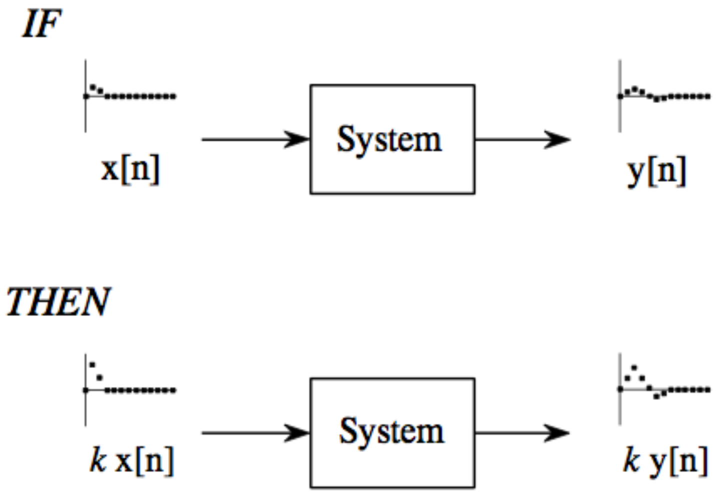
- What is a **signal**?
  - It can be regarded as a **description how** a **parameter varies** (dependent variable) **with respect to another** (independent variable);
  - E.g., the voltage of an electric charge varies with respect to time (1D signals) ;
  - E.g., the intensity of a pixel varies with respect its location in image (2D signals);
- Tipically, signals are denoted by **upper case letters**.
  - Discrete signals are denoted by **[]**:
    - E.g., X[n], Y[k]
  - Continuous signals are denoted by ()
    - E.g., X(i), Y(j)

Linear Systems

- A system is said to be **linear** if it complies two mathematical properties:
  - Homogeneity;
  - Additivity;
- There is a third property which is not a strict requirement for linearity, but it is mandatory for most pratical digital signal processing techniques:
  - Shift invariance

Linear Systems: Homogeneity

- Let f: R→R be a system, such that f(x)=y.
  - **If z=kx then f(z)=k f(x).**
- In practical terms a system is homogenous if an **amplitude change** in the **input** corresponds to an **identical amplitude** change in its **output**.



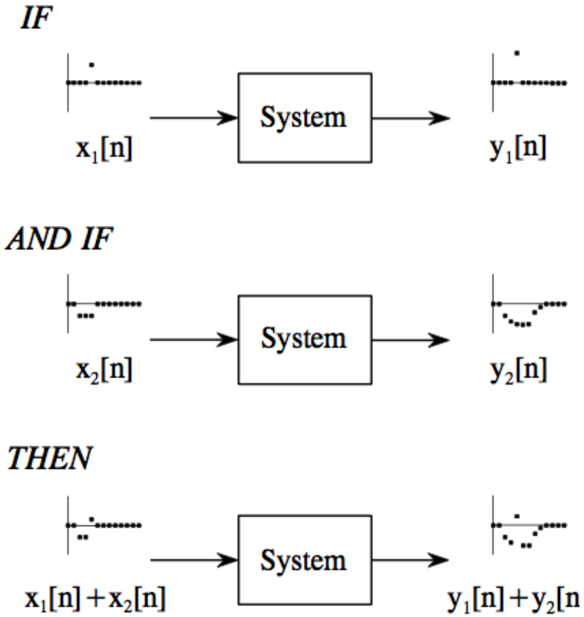
Linear Systems: Exercises

- Consider the following system $f: R^2 \rightarrow R$, such that:
  - $f(x,y)=2x-4y+2$
  - Determine the homogeneity of “f”.

- Now, consider the following system $g: R \rightarrow R$, such that:
  - $g(x)= \exp(x)$
  - Determine the homogeneity of “g”.

Linear Systems: Additivity

- Let f: R→R be one system, such that $f(x_1)=y$ and $f(x_2)=z$.
  - **If $x_3=x_1+x_2$ then $f(x_3)=f(x_1)+f(x_2)=y+z$**
- In practical terms a system is said to be **additive** if the added signals pass by the system **without interacting**.

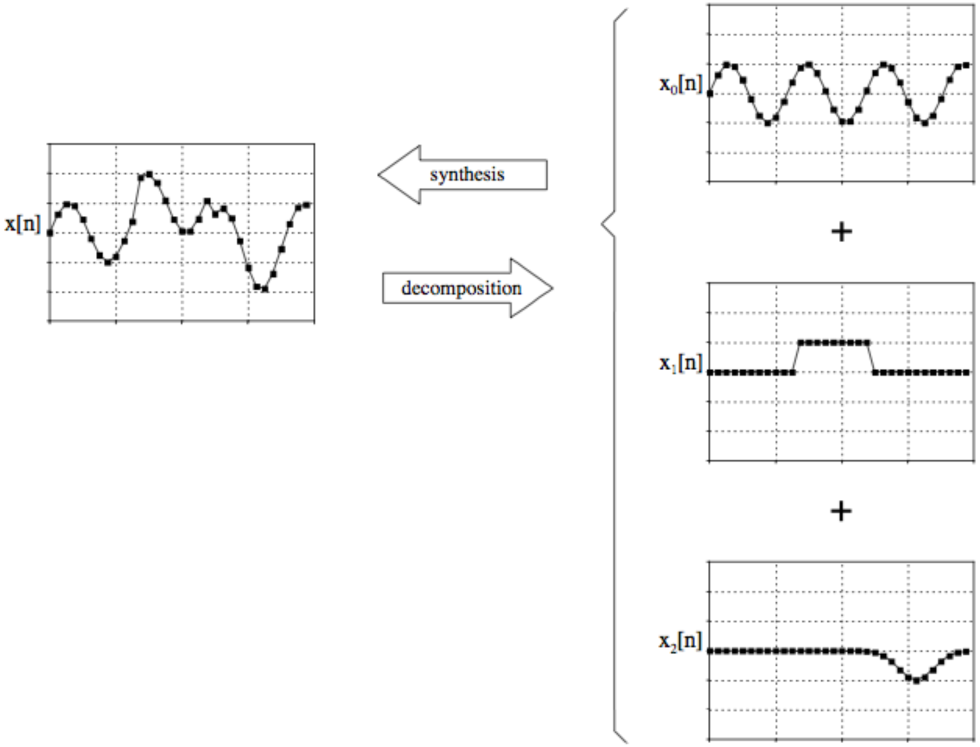


Linear Systems: Exercises

- Consider the following systems. Evaluate their additivity:
  - f: R→R, such that
    - f(x) = x;
  - g:R→R, such that
    - g(x)=0;
  - h:R^2→R, such that
    - h(x,y)=xy;
  - z:R^2→R, such that
    - z(x,y)=x+3y;

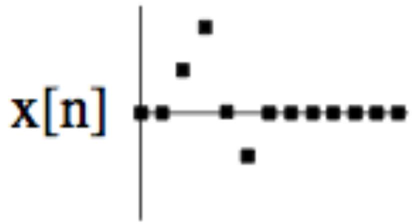
Superposition of Signals

- ❑When we are working with linear systems, the only way signals can be combined is by scaling (multiplication of the signals by constants), followed by addition.
  - ❑The process of combining several signals into a single one is called **synthesis**
  - ❑The inverse process, broking a signal into its fundamental parts, its called **decomposition**.

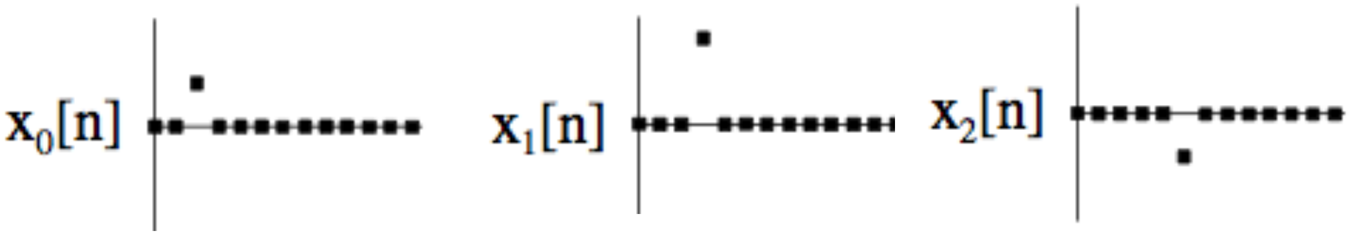


Superposition of Signals

- It's the **heart** of signal processing system.
  - It gives the overall strategy to understand how systems and signals are analyzed:
- Having one input signal:



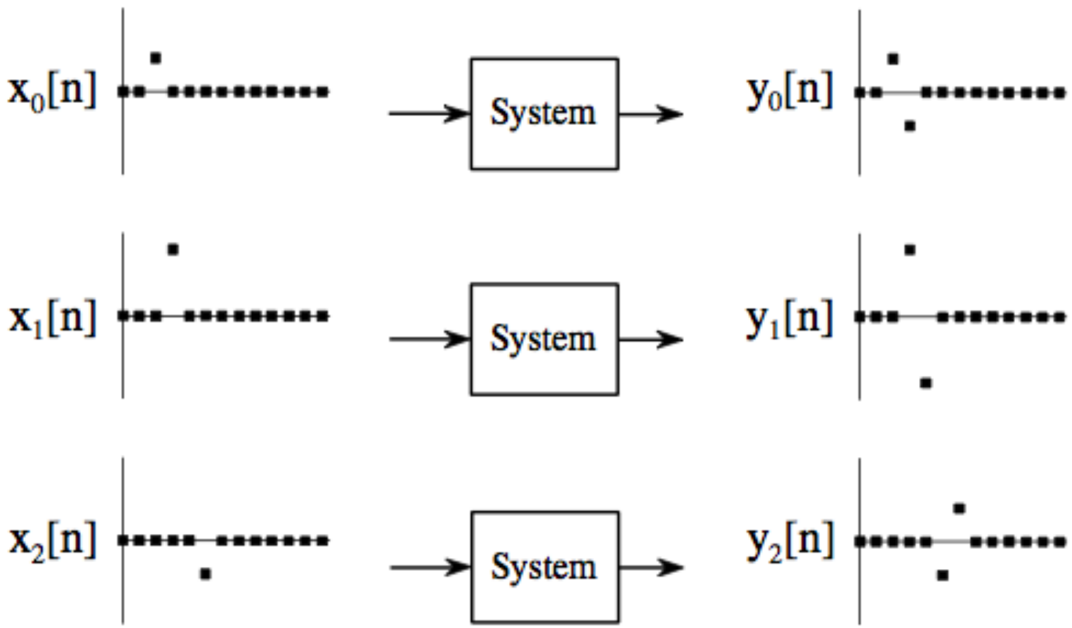
- We decompose it into simpler signals:



- ...remember that our goal is to **understand the system**!

Superposition of Signals

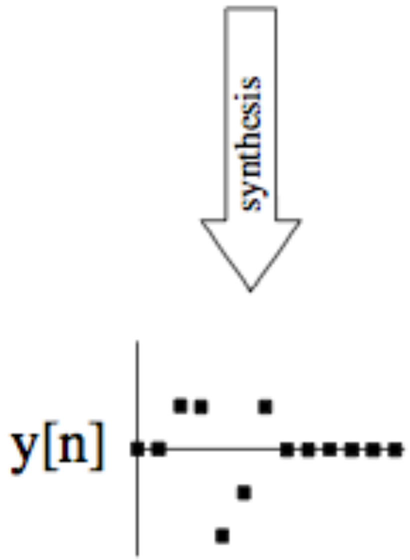
- Next, each input signal component passes individually through the system:



- These are the output signal components.

Superposition of Signals

- Instead of trying to understand how complicated signals pass through the system, all we need to know is how their **simplest components** are affected by the system.
- Finally, the output signal components are summed and we get the signal output, exactly equal as if the original signal was passed through the system.

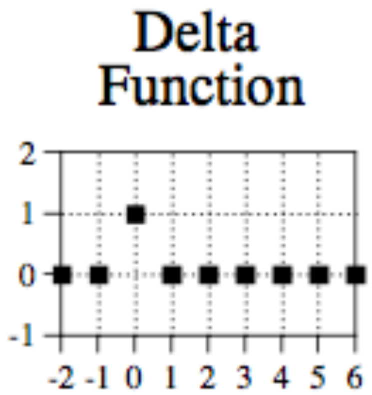


Signal Decomposition

- **Impulse Decomposition**
  - Decomposes the original signal “x” (length N) into N signals, where each component contains only one non-zero value:
    - $x_k(k)=x(k)$
    - $x_k(j)=0$, j<>k
- Impulse decomposition is important because it allows signals to be examined one sample at a time.
- By knowing how a system responds to an impulse, the system output can be calculated for any given input. This approach is called **convolution** and will be the subject of further discussions.
- **Exercise**: Consider the following signal, represented in time-domain:
  - [2,3,-4,1,0,5,2,4]
- Use impulse decomposition in the above signal and extract the resulting impulses.

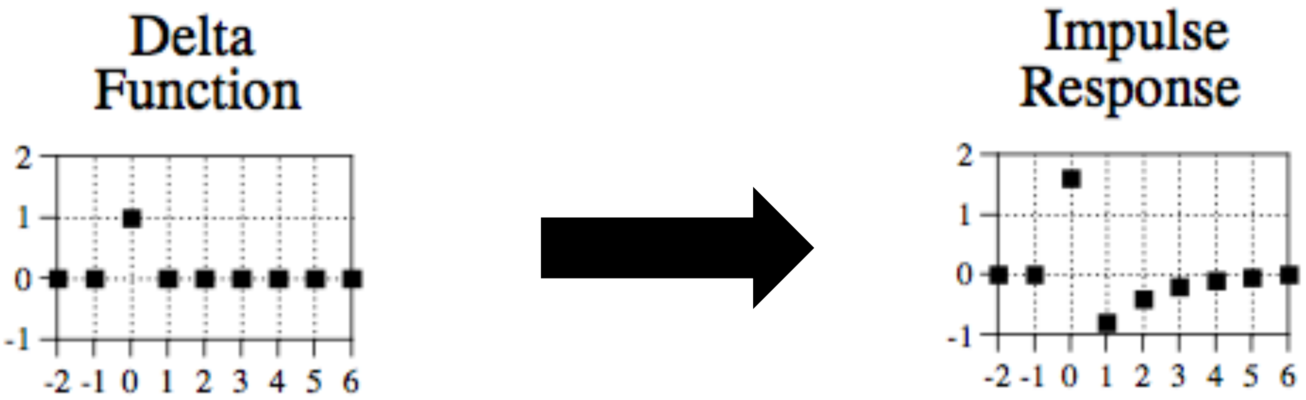
Signal Decomposition

- **Impulse Decomposition**
  - The notion of "**Delta function**" (d) is extremely important, when using impulse decomposition. A delta function has the central component equal to 1and the remaining ones equal to 0.
  - Let $f_k(x)$ be a signal resultant of input decomposition of f(x).
    - $f_k(x) = k\ d(x+t)$. Every input is s scaled and shifted version of the delta function

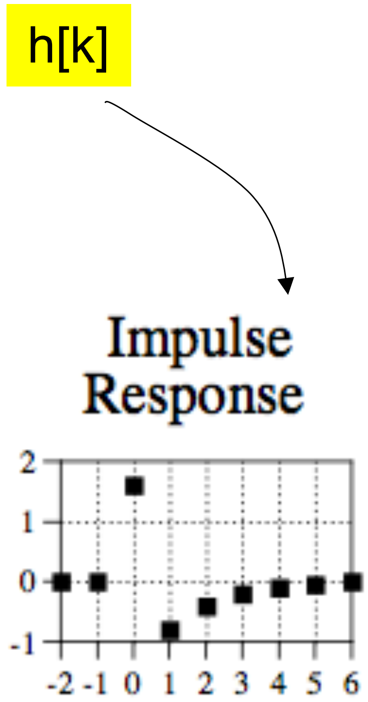


Inpulse Response

- **Impulse Decomposition**
  - According to the above discussion, the output signal can be found by adding the output of these scaled and shifted impulse responses.
  - In practical terms, if we know the response of a system to an impulse, we know the exact transformation that corresponds to our system.
    - Then, we can apply it to any arbitrary input signal.



Inpulse Response

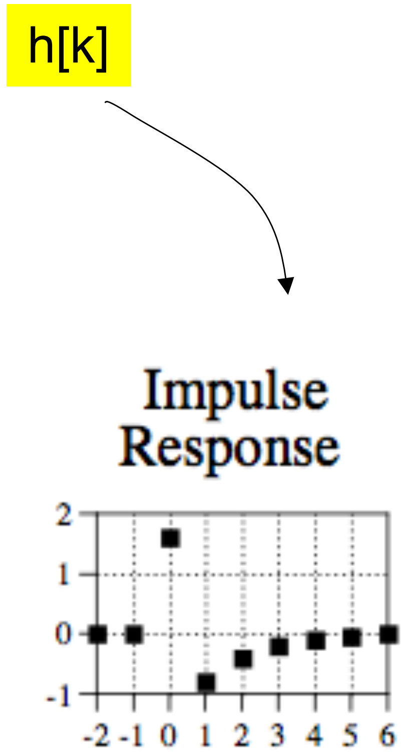


- **Impulse Decomposition**
  - The impulse response completely determines the system's behavior via **convolution.**

$$y[n] = \sum_{k=-\infty}^{\infty} h[k].x[n-k]$$

  - **In practice, the input response is the "kernel" that will be used to transform any input signal x[] in the corresponding output y[]**

Inpulse Response



Audio Processing:

- The impulse response of a **room (reverb kernel)** determines how a sound will reverberate.
- Convolution with this impulse response simulates different acoustic environments.

Image Processing:

- A **Gaussian blur** is applied by convolving an image with a Gaussian kernel (which acts as the impulse response).
- An **edge detection filter** uses an impulse response (like the Sobel or Laplacian kernel).

Communications & Control Systems:

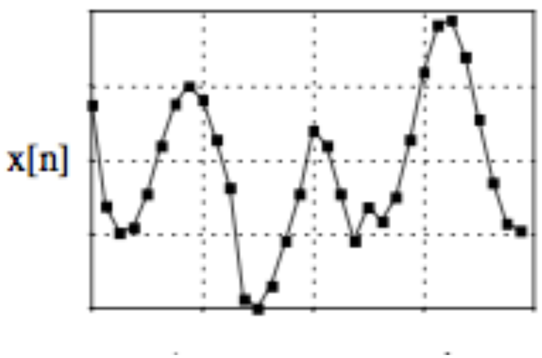
- In **wireless communication**, the multipath impulse response models how a transmitted signal arrives at a receiver after multiple reflections.
- In **control systems**, the impulse response helps design controllers by analyzing system stability.

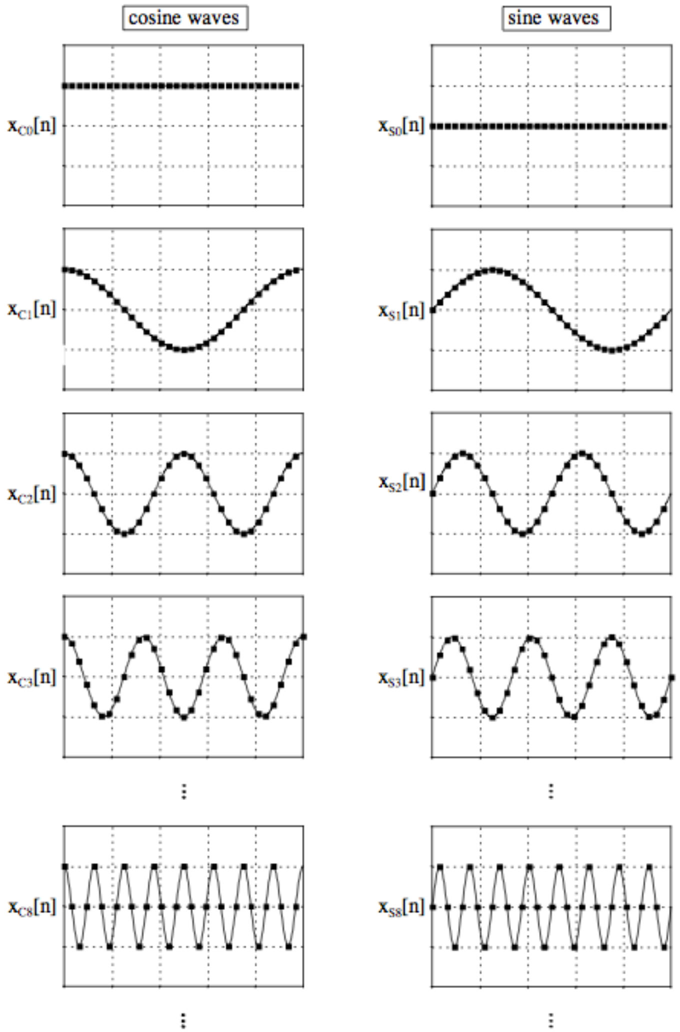
Signal Decomposition

❑ Fourier Decomposition

- ❑ It resulted from a very important finding, by J. Fourier
  - ❑ *"Any periodic signal can be decomposed by a (potentially infinite) sum of simpler periodic signals"*.
- ❑ In practice, it decomposes any N length signal into N+2 signals, half of them "sin" waves and the remaining "cosine" waves.
  - ❑ The first cosine component has fundamental frequency 0.
  - ❑ The second has fundamental frequency 1.
  - ❑ …
  - ❑ Similar observations for the sin waves.
- ❑ Since the frequency of each component is fixed, the only thing that changes for different signals being decomposed is the amplitude of each of the sine and cosine waves.
  - ❑ Hence, any signal can be represented by the amplitude of the corresponding "sin" and "cosine" waves.
  - ❑ Similar to a recipe, we can postulate that any input signal y[t] is a sum of "x_1" sin(0t) + "x_1" cos(0t) + …

Fourier Decomposition: Example





Signal Decomposition

- Exercise: Consider the following impulse response of a 1D signal in a system "f" (centered at index "0").
  - **[0, 0, -1, 0, 1, 0, 0]**
- Determine:
  - f([1,2,4,0,-1,1,0,2,3,1,0])

- In the general signal processing domain, the impulse response of a system is called "**filter kernel**" or "**convolution kernel**".
- In image processing, it is called point spread function.

Convolution

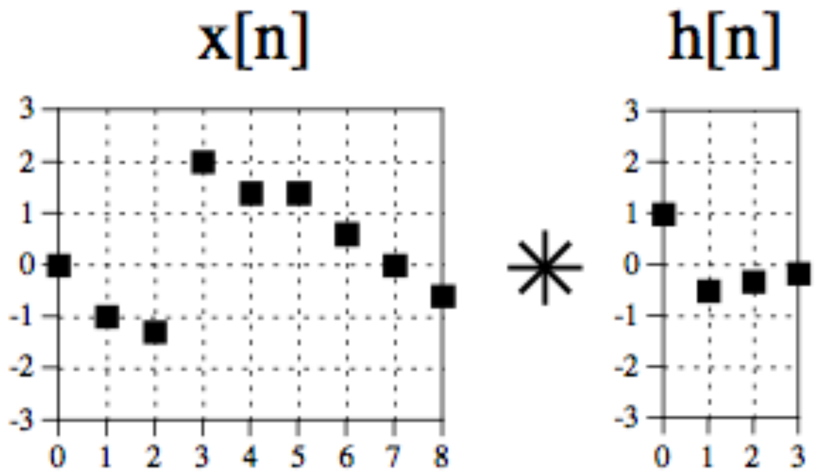
- It is a mathematical operation that describes the relationship between three signals:
  - One **input** signal;
  - One **impulse response;**
  - Yielding the **output** signal

- As it combines addition (+) with multiplication (x), it is usually denoted by “*****”.
  - Y[k]=H[k]*X[k]

$$y[i] = \sum_{j=0}^{M-1} h[j]\,x[i-j]$$

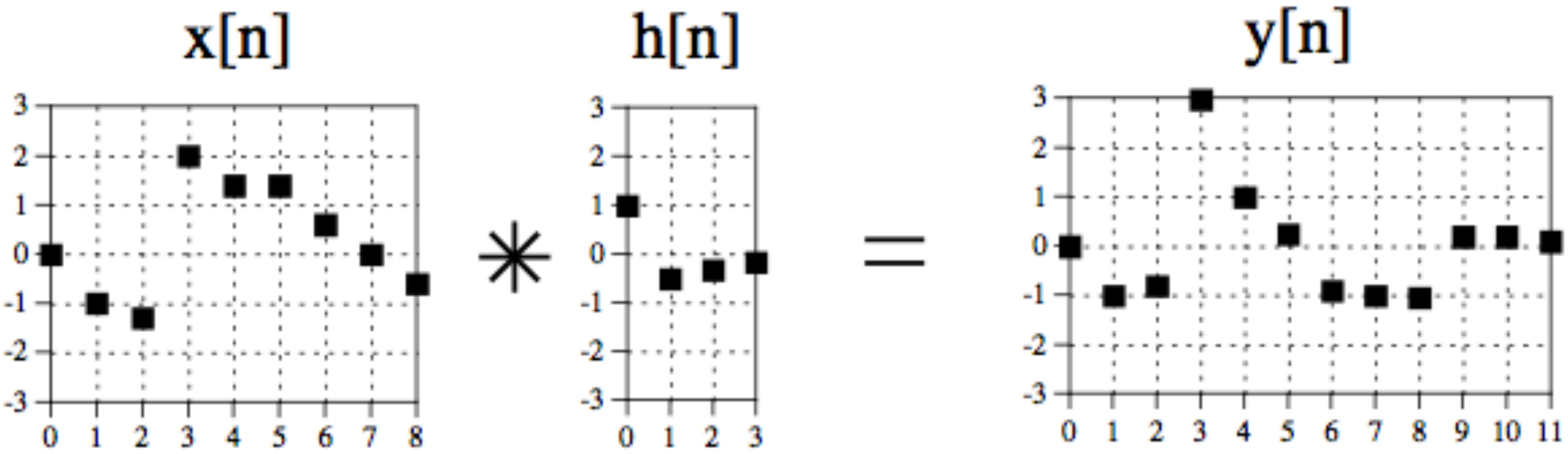
Convolution: Exercise

- Obtain the result of the convolution of the following signals:



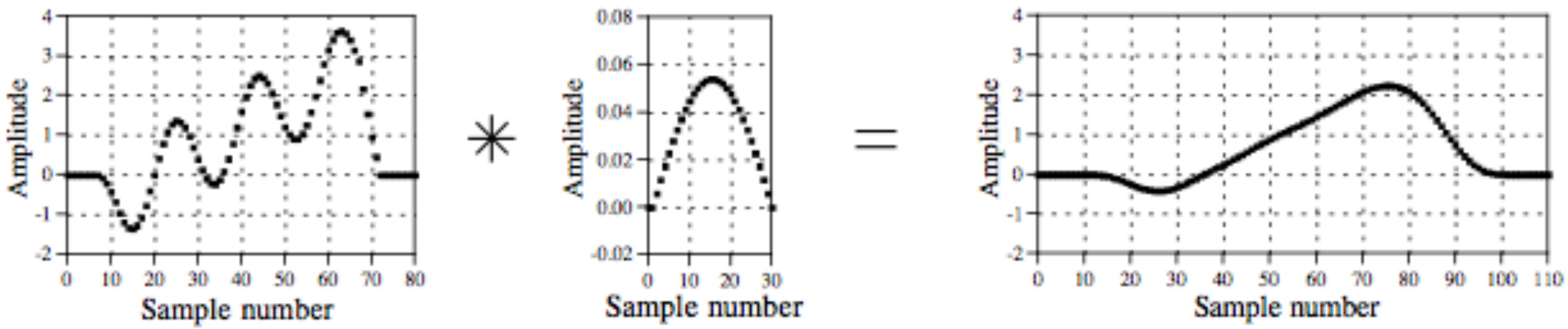
Convolution: Exercise

- Obtain the result of the convolution of the following signals:



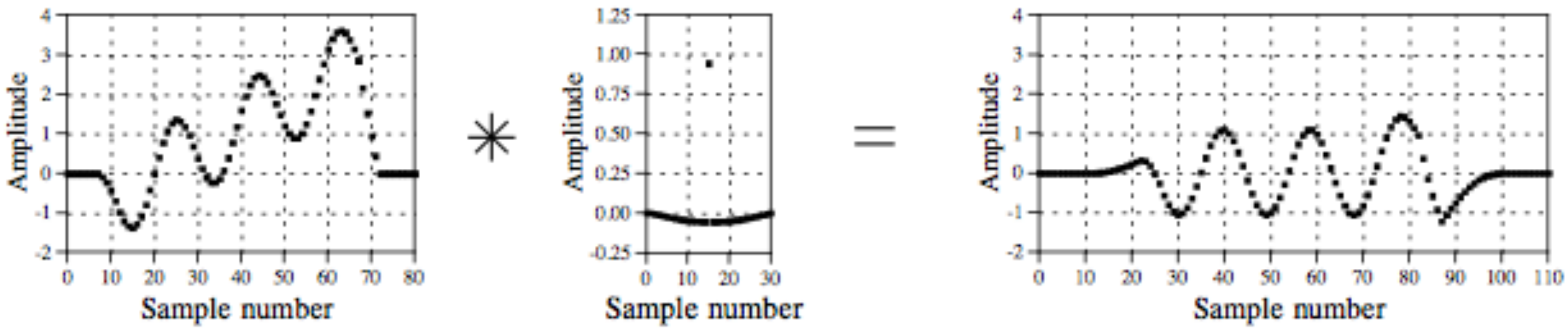
Convolution: Examples

❑**Low-pass filtering:**



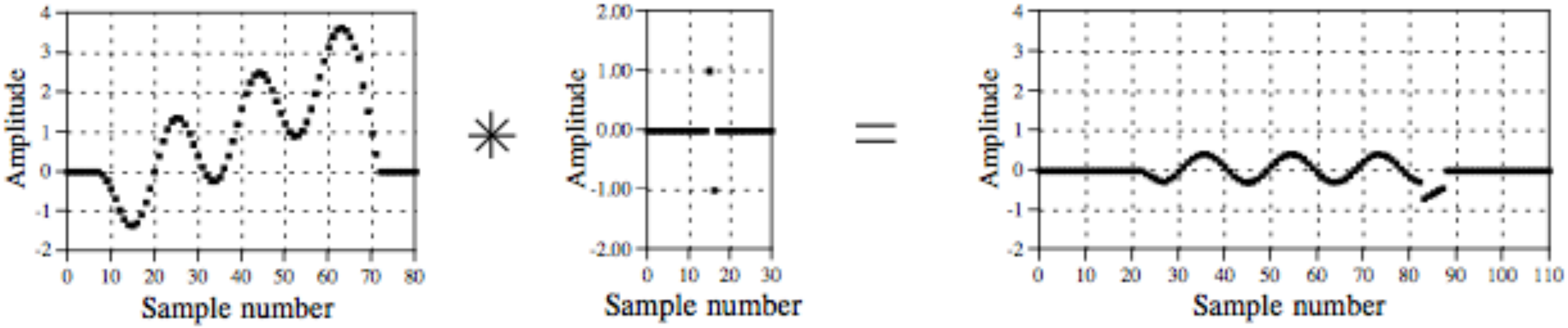
Convolution: Examples

❑**High-pass filtering:**



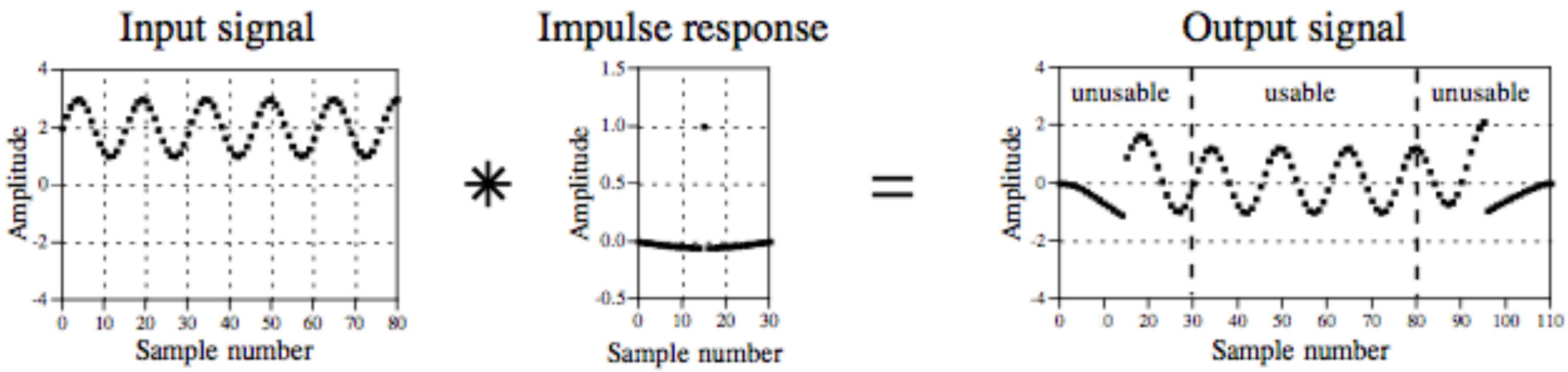
Convolution: Examples

❑Discrete derivative:



Convolution: Caution!!

- When an input signal is convolved with an impulse response of length "M", then the first and last "M-1" components are not fully reliable.
  - Why is this?



Frequency Domain

- Any signal can be represented by a linear combination of basis-functions.
- In case of 2D images, we have the following function:

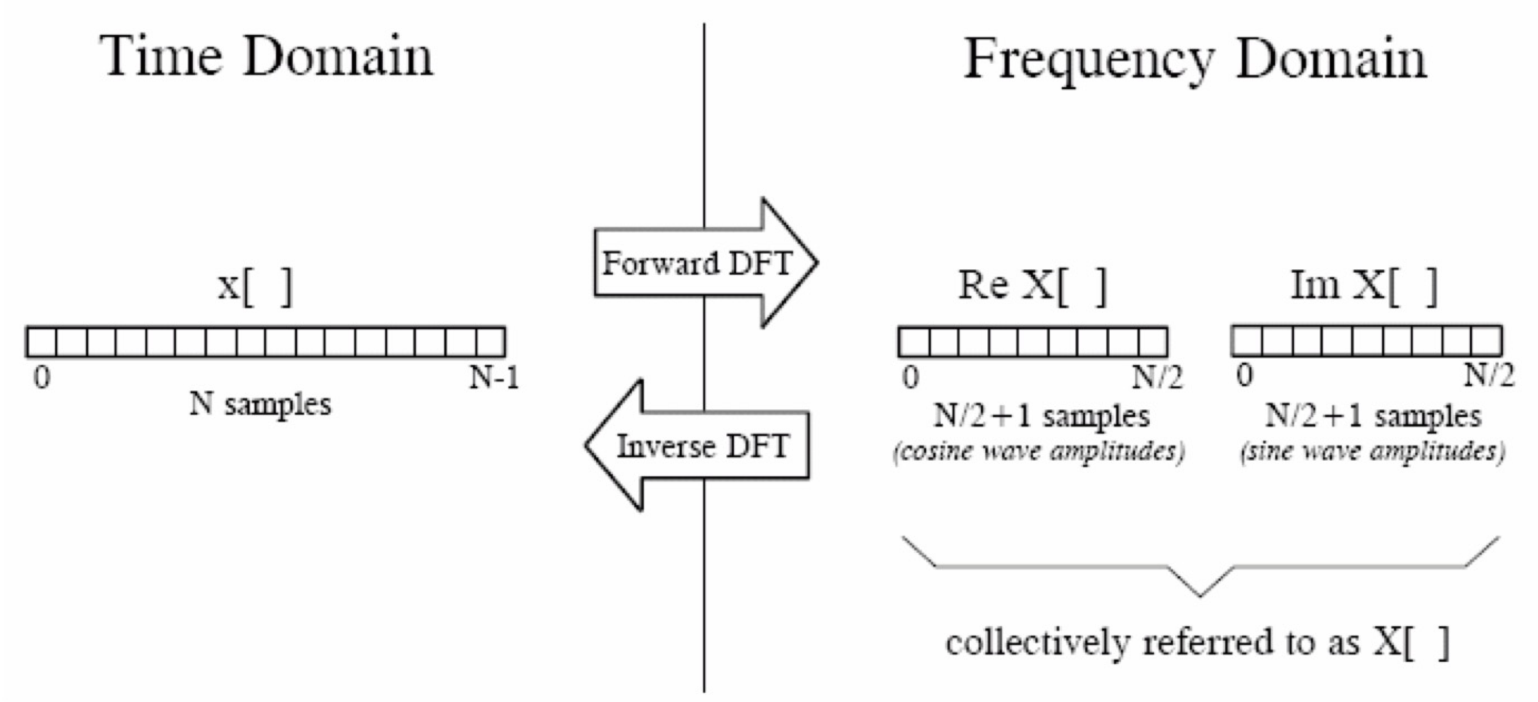
$$f(x, y) = \sum_k a_k \Psi_k(x, y)$$

- Here, a_k are the contributions of each basis-function to the original image.
- Basis functions are exponentials, complex and expressed in terms of harmonic functions ("*sin*" and "*cos*"):

$$\Psi_k(x, y) = \exp(i(\mu_k x + \nu_k y)) \qquad \exp(i\theta) = \cos(\theta) + i \sin(\theta)$$

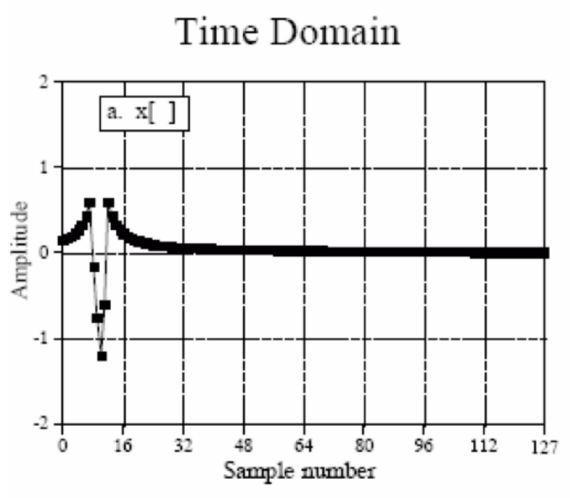
Discrete Fourier Transform (DFT)

- We can build the following correspondence between any signal represented in the time (space) and frequency domains:

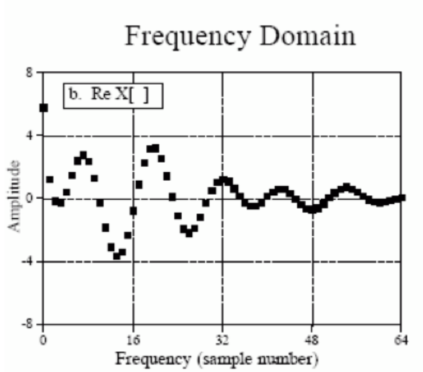


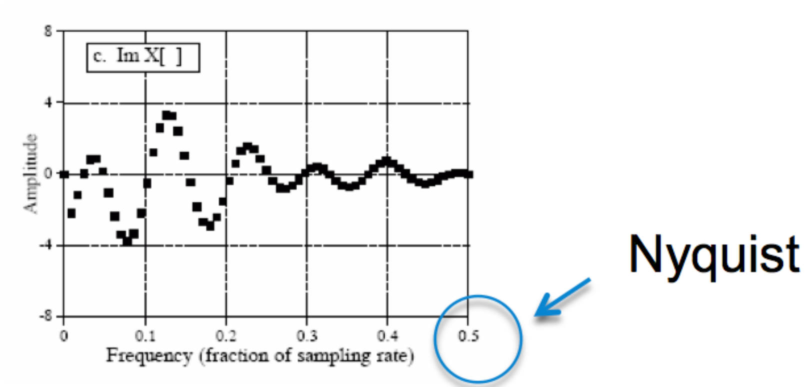
Discrete Fourier Transform (DFT)

❑Suppose we have the following signal, represented in the time-domain:



❑By using the DFT algorithm, we are able to express it in the following way:



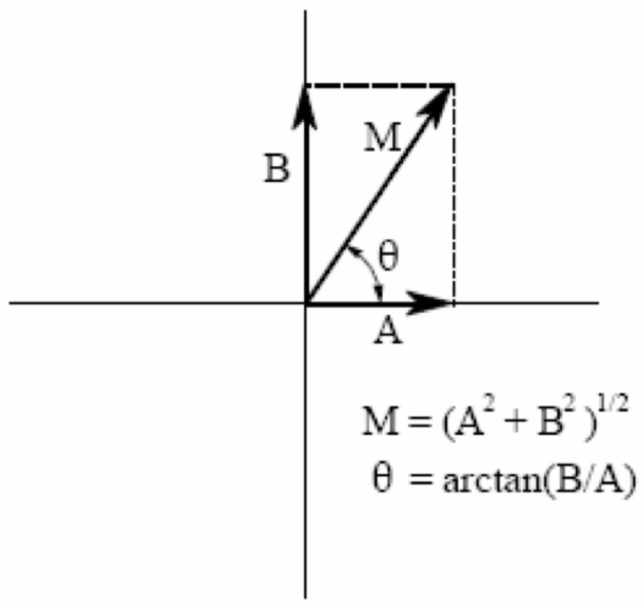


Discrete Fourier Transform (DFT)

- Often, it is more understandable to express the output of the DFT in Polar coordinates (**magnitude** + **phase**) , rather than in the original real and imaginary components:

  - Mag(X[k])
    - sqrt(Re(X[k])^2+Im(X[k])^2)

  - Phase(X[k])
    - arc tan(Im(X[k])/Re(X[k]))



Convolution Theorem

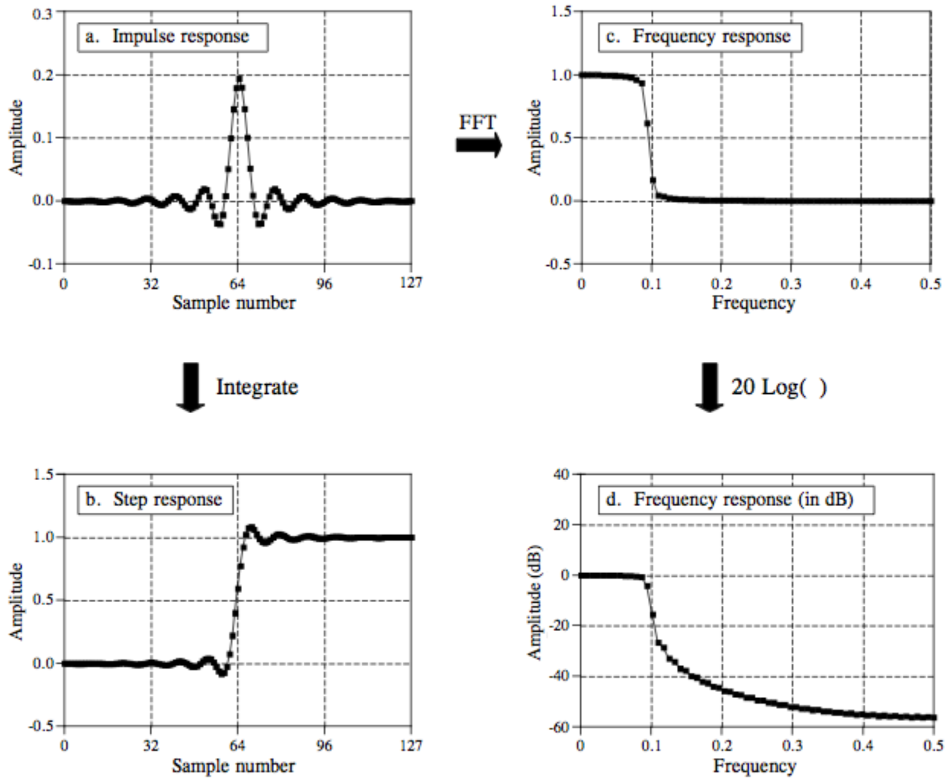
- The convolution of two signals in a given domain (either spatial or frequency) corresponds to the point-by-point multiplication in the complementary domain.

  - **H(x)=f(x)*g(x) ←→ H(x)=F(x) x G(x)**

- This is extremely important in modern DSP and in practical terms, enabled the existence of most state-of-the-art technologies and devices:
  - TV, radio, computer,…;

Filters

- ❑According to the convolution theorem, the convolution in time|frequency domain corresponds to multiplication in frequency|time domain.
  - ❑Each filter has an **impulse response**, a **step response** and a **frequency response**:



Filters

- **Impulse Response**
  - Output of the system to an impulse;

- **Step Response**
  - Output of the system when the input is a step;
    - It can be obtained without passing any signal thorugh the system.
    - By integrating (running sum in discrete mathematics) the impulse response.

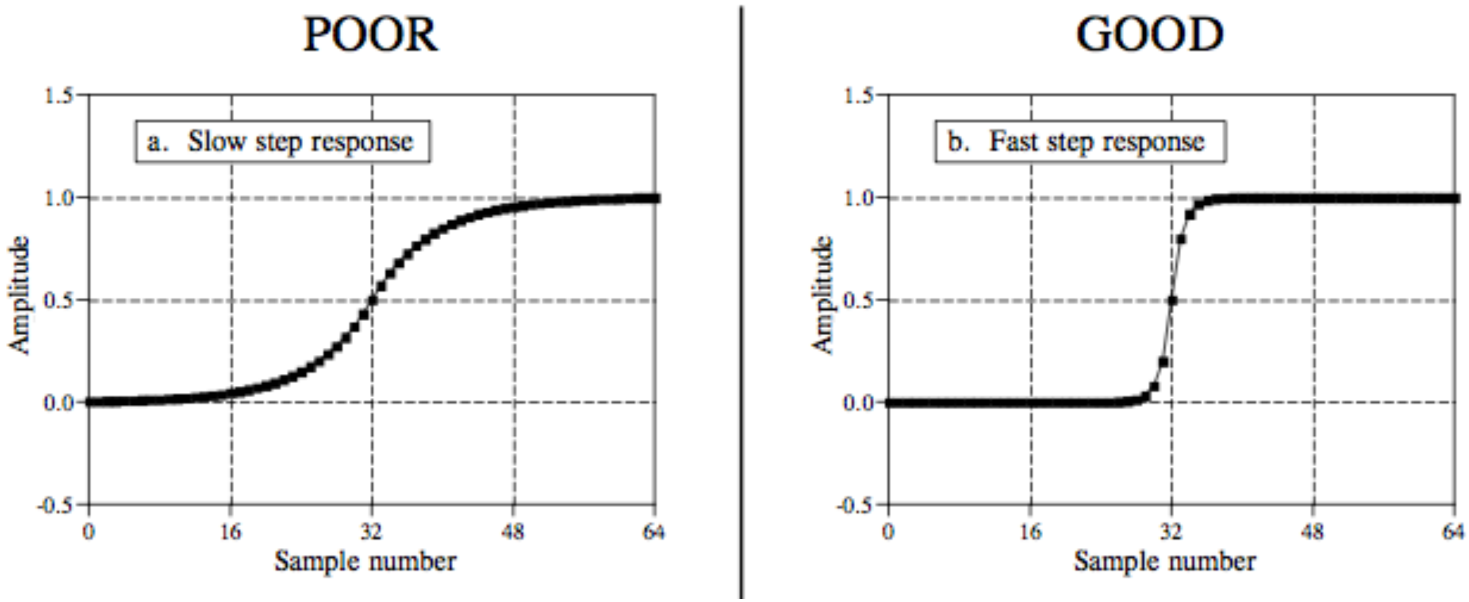
- **Frequency Response**
  - It can plotted in liner or logarithmic scales (decibels).
  - Corresponds to the **Fourier Transform** of the Inpulse Response

Filters

- The step response is often used to measure how well a filter performs in the time domain, mostly in terms of:
  - **Transition speed**. In order to discriminate components of a signal, the duration of the step should be shorter that the spacing of events. Thus, the transition speed should be as fast as possible.
    - Usually expressed by the proportion of samples between a low a high amplitude levels (10 and 90%).

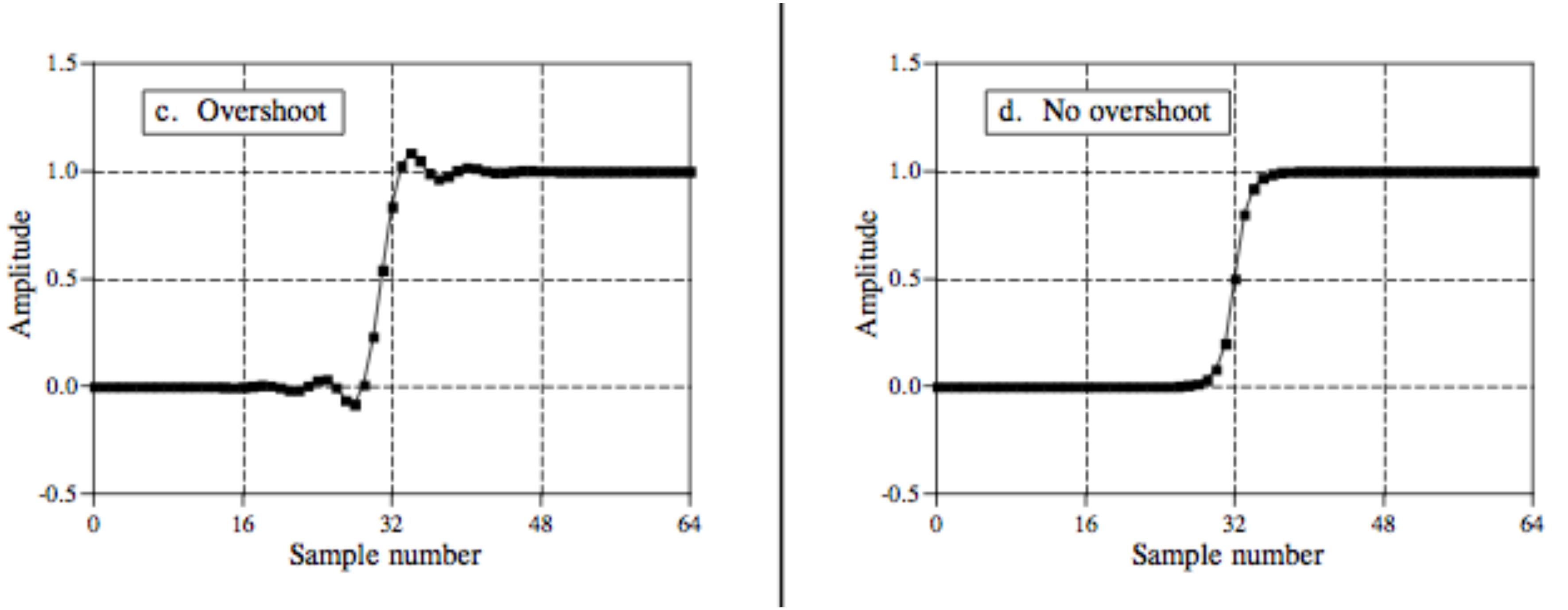
Filters: Examples

❑Transition Speed



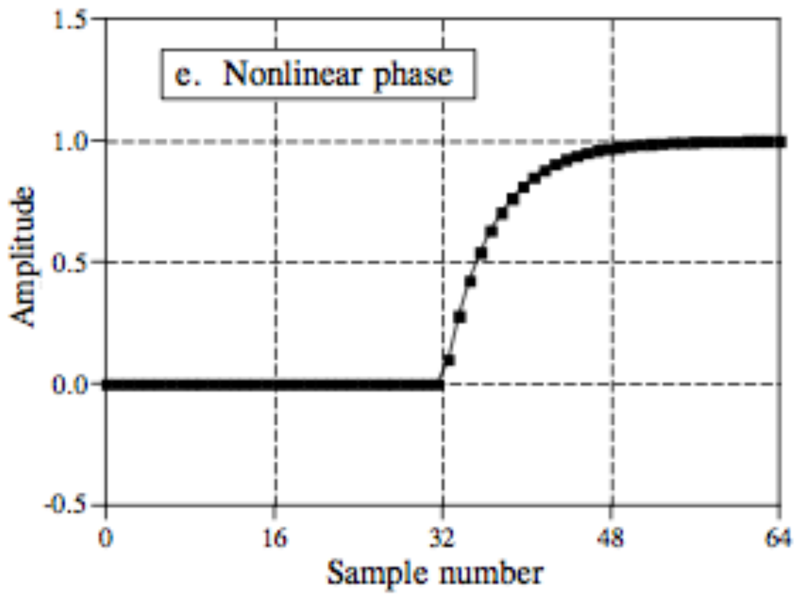
Filters: Examples

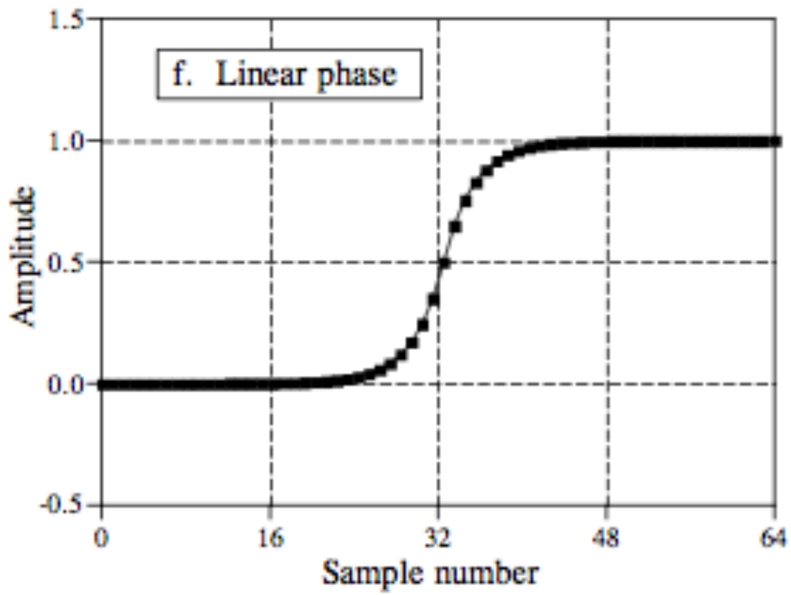
- **Overshoot.**
  - It corresponds to inverse variations to the major variation of step response.
  - It changes the signal amplitude non-homogeneously.



Filters: Examples

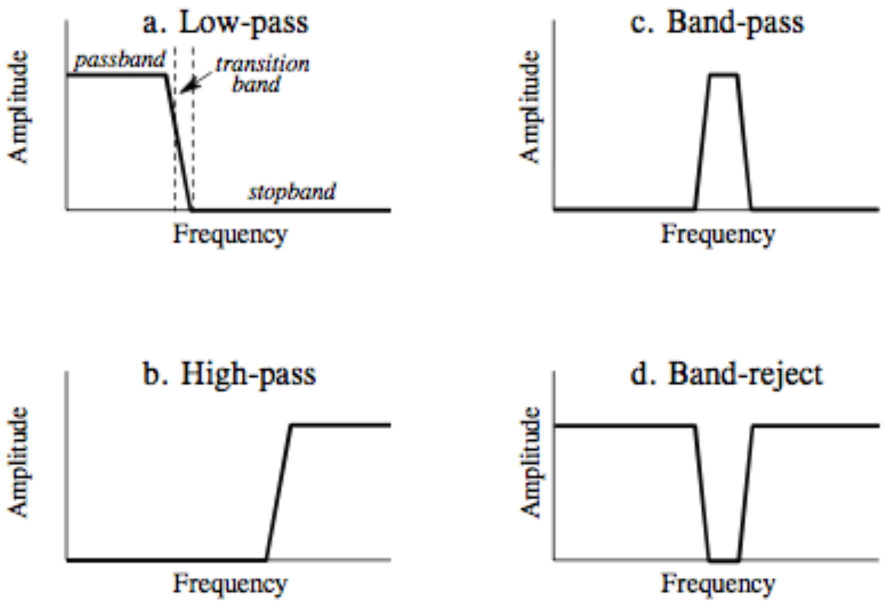
- Linear Phase.
  - Usually it is desired that the upper half of the step response is symmetrical to the lower half.





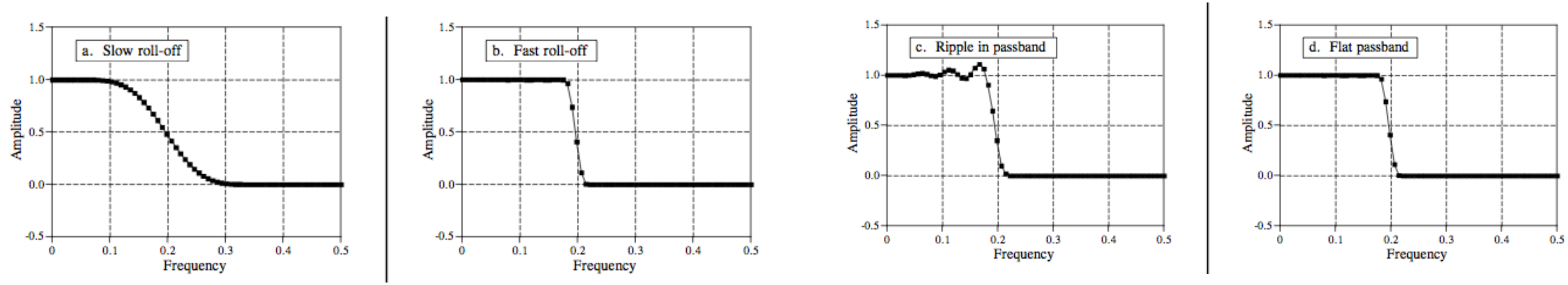
Filters: Summary

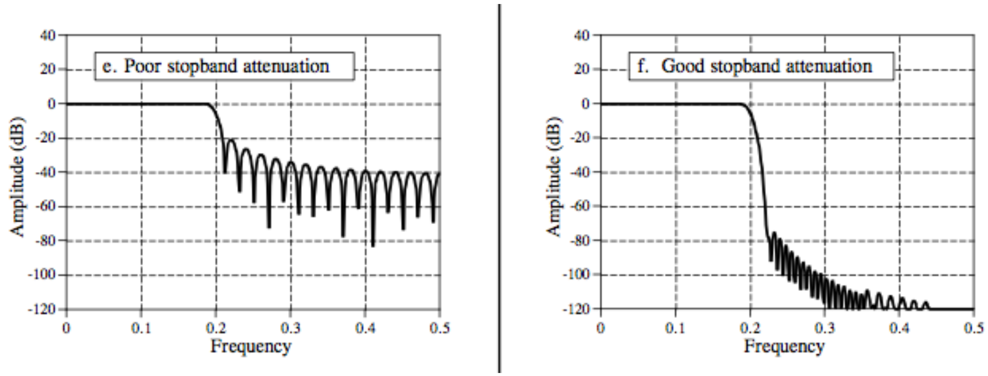
- When analyzing a system in terms of its frequency response, the most important factor is to observe the amount of frequencies that are blocked or passing through the system.
  - The **pass band** refers to the range of frequencies that pass trough the system
  - The **stop band** gives the frequencies that are blocked
  - The **transition band** is on the boundary



Filters

- We are usually interested in filters that have a fast **roll-off** (short transiction band) and without **ripples**. Finally, in order to actually block frequencies, we want to keep good **stop band attenuation** (expressed in logarithmic scale).

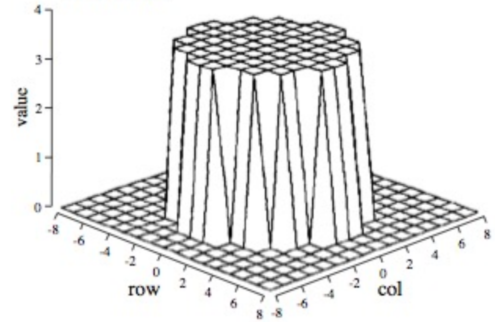




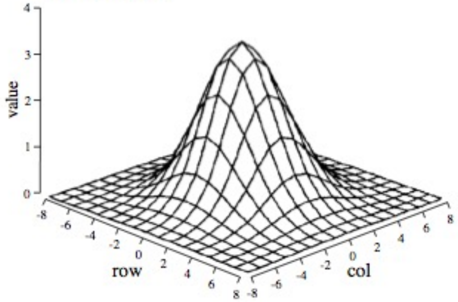
A Bel (Alexander Bell) expresses that the power is changed one order of magnitude. As such, decibel values of -10dB, 0dB, 10dB mean power ratios of 0.1, 1 and 10. **Amplitude is the square root of power**. As such, 20dB mean that amplitude changes one order of magnitude

Filters: Examples

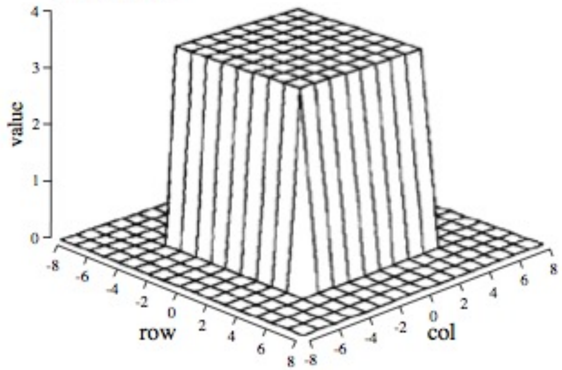
a. Pillbox



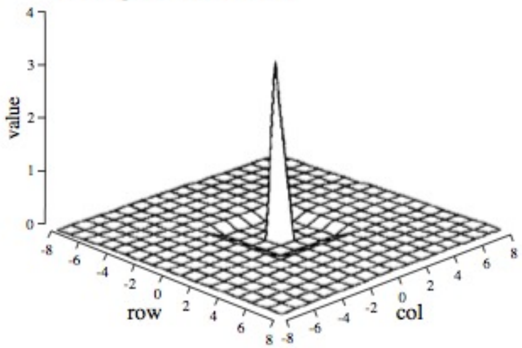
b. Gaussian



c. Square



d. Edge enhancement



e. Sinc